measurements, it is found that the number of unknowns in the problem exceeds the number of points for temperature measurement even for the simplest case z = 1. This case applies to the diode.

To reduce the number of unknowns, the experiment may be set up in such a way that $q_1(\tau) = q_2(\tau) \simeq 0$. Further, for a sufficiently thick crystal (~ 1 mm), it is possible to disregard the thickness of the heat-liberation region δ_i and to regard the discrete source as plane. The remaining three unknowns — the output per unit volume of the distributed sources $w_0(\tau)$, the output per unit volume of the discrete source $w_i(\tau)$, and its position x_i — may be determined by recording the surface-temperature variation of the crystal with time and the variation with time of the total losses in the crystal. A number of experimental procedures are possible. The next problem is to determine the thickness δ_i of the region in which the heat source acts. Another possibility to be investigated is the use of the method outlined in combination with the use of heat-sensitive parameters [7] requires further analysis.

The method of successive intervals may also be used to obtain an approximate three-dimensional picture. This involves the use of a diode with a cellular base. The crystal may then be considered as a collection of independent current tubes, and the temperature variation at several points of the crystal end surfaces may be determined experimentally. The use of the inverse problem allows the transverse distribution of the output per unit volume of the discrete sources in the crystal to be approximately determined.

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SOLUTION OF CONJUGATE PROBLEM IN SUCCESSIVE INTERVALS

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UDC 536.24

The conjugate problem is solved for nonsteady heat transfer through a plane wall with convective heat transfer at the edges.

On the basis of experimental data on the startup conditions of heat transfer [1, 2], the energy equation for the thermal boundary layer and the heat-conduction equation for the wall are solved jointly. Having obtained the solution, the variation with time in the temperature of the heat-transfer surfaces and the heat flows in the course of nonsteady heat transfer may be determined for given parameters.

In formulating the problem it is assumed that the flow of liquid is stable, the flow rate is given, and its mean velocity over the cross section is known. The liquid-flow temperature is assumed to be constant and equal to the liquid temperature at the inlet to the heat-transfer section. The liquid is incompressible with constant thermophysical properties. Energy dissipation due to viscosity and heat conduction of the wall material in the longitudinal direction of liquid flow is neglected. The mean heat-transfer coefficient is referred to the difference between the temperature of the heat-transfer surface of the wall and the liquid temperature.

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TABLE 1. Coefficients of Eq. (3)

Experiment No.	Material	Re_h	Re_c	C_1	B_1	$q(0, 0)$	C_2	B_2
1	Co	1800	1810	1,4026	432,3	29683	2,2546	1169
2	Ag	1815	1790	1,3051	403,3	27697	2,2546	1165
3	Co	1800	3360	1,4026	432,3	29683	2,771	1766
4	Ag	1815	3320	1,2074	373,1	25620	2,221	1410
5	Co	4450	1810	2,785	1160,5	42251	2,2546	1169
6	Ag	4500	1810	2,44	1020,6	37156	2,2546	1169
7	Co	7150	1810	4,00	1952,3	71076	2,2546	1169
8	Ag	7150	1790	3,62	1766,8	64325	2,2546	1169
9	Co	7150	3360	4,00	1952,3	71076	2,771	1766
10	Ag	7150	3320	3,62	1766,8	64325	2,387	1515

The joint solution of the energy equation for a thermal boundary layer and the heat-conduction equation for the solid wall will be represented as a combination of independent solutions satisfying boundary conditions of the fourth kind at the contact plane between the liquid and the solid wall.

In solving the energy equation for a thermal boundary layer, it is assumed that the velocity distribution in the liquid flow corresponds to the Hagen—Poiseuille law, and the temperature distribution is described by a cubic parabola. The solution of the energy equation written in integral form [3] determines the heat flux:

$$q_{w(x)} = 0.84\,(t_f - t_w)\left[\frac{(\rho c_p)_f \bar{w}\lambda_f^2}{rx}\right]^{1/3}.$$

Hence the mean heat-transfer coefficient is determined in the form

$$\bar{\alpha} = 1.26\left[\frac{(\rho c_p)_f \lambda_f^2 \bar{w}}{rl}\right]^{1/3},$$

or in the form of a dimensionless number

$$\overline{\mathrm{Nu}}_d = 1.5876\left(\mathrm{Re}_d \mathrm{Pr}_f \frac{d}{l}\right)^{1/3}. \tag{1}$$

Using this method of solving the energy equation for other velocity and temperature distributions in the flow leads to analogous solutions that differ in the value of the constant coefficient. A similar solution has been used [4] to determine the mean value of Nu in the initial thermal region of a plane tube with constant wall temperature. For nonsteady heat transfer it is impossible to predict in advance the form of the temperature and velocity distribution in the flow and so it is expedient to write Eq. (1) in the general form

$$\overline{\mathrm{Nu}} = C\left(\mathrm{Re}_d \mathrm{Pr}_f \frac{d}{l}\right)^{1/3}. \tag{2}$$

Analysis of the experimental data of [1, 2] shows that, for nonsteady heat transfer at a heated surface, the heat-transfer coefficient in laminar flow changes in proportion to $(\mathrm{Pr}^*)^{1/3} = (\mathrm{Pr}_w/\mathrm{Pr}_f)^{1/3}$, whereas in transient flow conditions the inverse relation holds: $(\mathrm{Pr}^*)^{1/3} = (\mathrm{Pr}_f/\mathrm{Pr}_w)^{1/3}$. On a cooled surface of the wall, the heat-transfer coefficient remains constant; i.e., $\mathrm{Pr}^* = \mathrm{Pr}_f$.

Taking into account the experimental relations, Eq. (2) takes the form

$$\overline{\mathrm{Nu}}_d = C\left(\mathrm{Re}_d \mathrm{Pr}^* \frac{d}{l}\right)^{1/3}. \tag{3}$$

The solution of the heat-conduction equation in successive intervals with variable step for a piecewise-linear approximation of asymmetric boundary conditions of the second kind and arbitrary initial temperature distribution is given in [5].

The solution of the system of equations consisting of two equations for the temperature of the heat-transfer surfaces [5] and the two relations in Eq. (3), resolved with respect to the heat fluxes (taken, respectively, for the two heat-transfer surfaces of the wall), is as follows:

$$t(0, \mathrm{Fo}_k) = \left\{\left[1 + B_2 \frac{R}{\lambda_w} A(0, \omega_k)\right] q^*(0, \mathrm{Fo}_k)\right.$$

$$- B_2 \frac{R}{\lambda_w} A(1, \omega_k) q^*(1, \mathrm{Fo}_k) + B_2 A(1, \omega_k) t_{f_2}$$

$$+ \left\{ B_1 B_2 \frac{R}{\lambda_w} [A^2(0, \omega_k) - A^2(1, \omega_k)] + B_1 A(0, \omega_k) \right\} t_{f_1} \mathrm{Pr}^{*1/3} \Bigg\} \tag{4}$$

$$\times \left[\frac{\lambda_w}{R} + B_2 A(0, \omega_k) + \left\{ B_1 A(0, \omega_k) + B_1 B_2 \frac{R}{\lambda_w} [A^2(0, \omega_k) - A^2(1, \omega_k)] \right\} \mathrm{Pr}^{*1/3} \right]^{-1},$$

$$t(1, \mathrm{Fo}_k) = \left\{ q^*(1, \mathrm{Fo}_k) + B_2 A(0, \omega_k) t_{f_2} + B_1 A(1, \omega_k) [t_{f_1} \right.$$

$$\left. - t(0, \mathrm{Fo}_k)] \mathrm{Pr}^{*1/3} \right\} \left[\frac{\lambda_w}{R} + B_2 A(0, \omega_k) \right]^{-1}, \tag{5}$$

where

$$q^*(0, \mathrm{Fo}_k) = \frac{\lambda_w}{R} L(0, \mathrm{Fo}_k) + q(0, 0) E(0, \mathrm{Fo}_k)$$

$$- q(1, 0) E(1, \mathrm{Fo}_k) + \sum_{i=1}^{k-1} q(0, \mathrm{Fo}_i) \Gamma(0, \mathrm{Fo}_i^*) - \sum_{i=1}^{k-1} q(1, \mathrm{Fo}_i) \Gamma(1, \mathrm{Fo}_i^*),$$

$$q^*(1, \mathrm{Fo}_k) = \frac{\lambda_w}{R} L(1, \mathrm{Fo}_k) + q(0, 0) E(1, \mathrm{Fo}_k) - q(1, 0) E(0, \mathrm{Fo}_k)$$

$$+ \sum_{i=1}^{k-1} q(0, \mathrm{Fo}_i) \Gamma(1, \mathrm{Fo}_i^*) - \sum_{i=1}^{k-1} q(1, \mathrm{Fo}_i) \Gamma(0, \mathrm{Fo}_i^*),$$

$$B_1 = C_1 \lambda_{f_1} \left(\frac{\mathrm{Re}_{d_1}}{l d_1^2} \right)^{1/3}, \quad B_2 \equiv \bar{\alpha}_2 = C_2 \lambda_{f_2} \left(\frac{\mathrm{Re}_{d_2} \mathrm{Pr}_{f_2}}{l d_2^2} \right)^{1/3}.$$

The functions A, E, and Γ are defined and tabulated in [5].

Having solved the conjugate problem in Eqs. (4) and (5) it is possible to determine the temperature of the heat-transfer surfaces for given values of C_1, C_2, t_{f1}, t_{f2}, t_0, R, and λ_W in successive intervals in the course of nonsteady heat transfer. Simultaneously, the heat fluxes $q(0, \mathrm{Fo}_k)$ and $q(1, \mathrm{Fo}_k)$ and the heat-transfer coefficient $\alpha(0, \mathrm{Fo}_k)$ at the corresponding moment of time are determined. In the first interval, $t(0, \mathrm{Fo}_1)$ is first determined from the value of Pr_W at the initial temperature $t(0, 0)$. From the value of $t(0, \mathrm{Fo}_1)$ obtained, a more accurate value of Pr_W is found and iterative recalculation of $t(0, \mathrm{Fo}_1)$ is begun. The last value of Pr_W in the first interval is the initial value for the calculation in the second interval and so on until the final moment of time.

It is easy to obtain the solution of the conjugate problem in the special case when the temperature of one of the heat-transfer surfaces is known or when one of the sides of the wall is heat insulated.

The solution of the conjugate problem has been used to calculate the variation of the temperatures of the heat-transfer surfaces t(0, Fo) and t(1, Fo), the heat fluxes q(0, Fo) and q(1, Fo), and the heat-transfer coefficients $\alpha(0, \mathrm{Fo})$ and $\alpha(1, \mathrm{Fo})$ in startup conditions of heat-transfer for two wall materials. The wall parameters adopted are as follows: $t(0, 0) = 25°C$, $R = 0.05$ m, $l = 0.015$ m, $\lambda_W(\mathrm{Co}) = 71.176$ W/m·°C, λ_W (Ag) = 418.68 W/m·°C, $(c\rho)_{Co} = 4024.3$ kJ/m³·°C, $(c\rho)_{Ag} = 2461.8$ kJ/m³·°C; the corresponding liquid-flow parameters are: $t_{f_h} = 75°C$, $t_{f_c} = 25°C$, $d_h = 0.0347$ m, $d_c = 0.0344$ m, $\lambda_{f_h} = 0.6647$ W/m·°C, $\lambda_{f_c} = 0.6065$ W/m·°C, $\mathrm{Pr}_{f_h}^{1/3} = 1.332$, $\mathrm{Pr}_{f_c}^{1/3} = 1.830$, where the subscripts h and c denote hot and cold flows, respectively.

Values of the constant coefficients in Eq. (3) calculated from the experimental data are shown in Table 1.

Comparison of experimental and calculated results for the temperatures of the heat-transfer surfaces and the heat fluxes in the course of nonsteady heat transfer shows that in all the experiments the maximum discrepancy between experiment and calculation does not exceed 3-4%.

Calculated values of the heat-transfer coefficient as a function of the temperature head $\alpha(0, \vartheta)$ and $\alpha(1, \vartheta)$ are shown in Fig. 1; the continuous lines show data for the cobalt wall and the dashed lines data for silver.

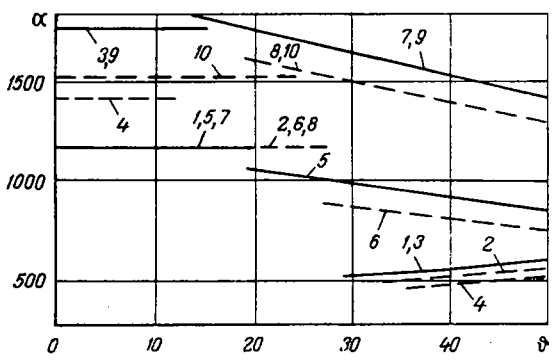

Fig. 1

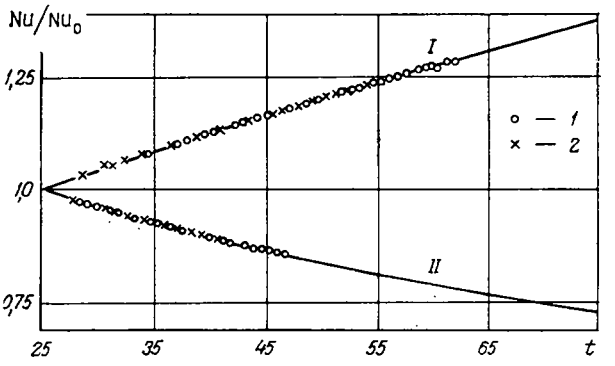

Fig. 2

Fig. 1. Dependence of heat-transfer coefficient on temperature head (calculated results): the numbers on the curves refer to the experiment No. α, W/m^2·°C; ϑ, °C.

Fig. 2. Dependence of reduced heat-transfer coefficient on surface temperature: I) transient conditions, $(Pr_0/Pr_W)^{1/3}$; $Nu_0 = C_1 (Re_d Pr^* \cdot d/l)^{1/3}$; $Pr^* = Pr_f/Pr_0$; II) laminar flow, $(Pr_W/Pr_0)^{1/3}$; $Pr^* = Pr_c/Pr_f$; Pr_0 corresponds to $t_0 = 25°C$; 1) Co; 2) Ag.

The variation in C_1 as a function of Re_h for a given wall material is in good agreement with the linear dependence $C_1 = C_0 + k_W Re_h$, where $C_0 = 0.55$, $k_W(Co) = 0.49 \cdot 10^{-3}$, $k_W(Ag) = 0.4242 \cdot 10^{-3}$. The slope coefficient varies in proportion to the cube root of the bulk specific heat of the wall material.

Experimental results for startup conditions of nonsteady heat transfer, generalized with respect to the variation in the reduced heat-transfer coefficient (the ratio of $\overline{Nu}_{Fo}$ to the initial value $\overline{Nu}_0$), are shown in Fig. 2 as a function of the temperature of the heat-transfer surface: for laminar flow, $A = (Pr_W/Pr_0)^{1/3}$; for transient flow, $A = (Pr_0/Pr_W)^{1/3}$. Here Pr_0 corresponds to the initial temperature and Pr_W to the current temperature (including the steady value) of the heat-transfer surface. Regardless of the wall material, all the experimental data for different conditions of heat-transfer-agent flow were in good agreement with the ratios of the reduced heat-transfer coefficient.

For laminar flow in the absence of natural convection (as in the experiment) heat transfer normal to the direction of motion is by heat conduction; then, neglecting the variation in the kinematic viscosity of the liquid with increase in temperature of the heat-transfer surface, it follows that

$$Pr^* = \frac{Pr_w}{Pr_f} \simeq \frac{a_f}{a_w}. \tag{6}$$

For turbulent flow (including transient conditions), there is additional heat transfer in this direction as a result of pulsations (convection) and the pulsational transfer considerably exceeds the heat transfer by conduction. In this case, neglecting the variation in the thermal diffusivity of the liquid with increase in the temperature of the heat-transfer surface, it follows that

$$Pr^* = \frac{Pr_f}{Pr_w} \simeq \frac{\nu_f}{\nu_w}. \tag{7}$$

For water the thermal diffusivity increases with increase in temperature and hence, according to Eq. (6), Pr* decreases in laminar flow; the kinematic viscosity decreases and, according to Eq. (7), Pr* increase in transient conditions. In accordance with the variation in Pr*, the heat-transfer coefficient also changes: increase in temperature of the heat-transfer surface leads to decrease in α in laminar flow and to increase in α in transient conditions, in accordance with experimental results.

Note, in conclusion, that solving the conjugate problem in successive intervals on the basis of the quasisteady approximation, taking into account the change in Pr*, leads to a description of convective heat transfer and the identification of a number of laws determining the value of the heat-transfer coefficient.

NOTATION

x, coordinate, m; τ, time, sec; t, temperature, °C; ϑ, temperature head, °C; ρ, density, kg/m^3; c, specific heat, J/kg·°C; λ, thermal conductivity, W/m·°C; ν, kinematic viscosity, m^2/sec; q, specific heat

flux, W/m^2; α, heat-transfer coefficient, $W/m^2 \cdot °C$; w, velocity of heat-transfer-agent flow, m/sec; d,r, equivalent diameter and radius of channel, m; l, length of heat-transfer section, m; R, determining dimension (thickness) of all, m; Nu, Re, Pr, Fo, Nusselt, Reynolds, Prandtl, and Fourier numbers; L, initial temperature distribution function; A, E, Γ, dimensionless functions; C, B, constants. Indices: f, w, fluid (liquid) and wall; 0, initial value; 1, 2, heat-transfer surfaces; k, i, calculational and current time intervals.

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ALGORITHM FOR CALCULATING TEMPERATURE FIELDS IN THIN-WALLED STRUCTURAL ELEMENTS

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An algorithm for calculating temperature fields in thin-walled structural elements is considered which is based on the concept of local one-dimensional schemes in conjunction with graphical solution of problems in heat conduction.

In investigating the thermal regime of various structures, one often encounters the problem of calculating temperature distribution in support elements having complex configuration as a rule. To calculate the temperature distribution in these elements, the method of finite elements, which is based on a study of the thermal balance in the elementary volumes into which an element is divided, is the method mainly used. Calculation of the thermal balances in the selected volumes is a laborious and tedious problem for which the solution is of a specific nature in each case.

An attempt was made [1] at universalization of the methods for computing multidimensional temperature fields in structures. The method discussed in that paper finds application in the investigation of temperature fields of various structures whose elements are of relatively simple configuration. In the case of individual elements of nontrivial shape, it is still necessary to use the approach of [1] to calculate the temperature fields in such elements and this complicates the problem. In order to construct relatively simple methods for investigating the thermal regime of individual elements, this paper considers an algorithm for calculating temperature fields in thin-walled structural elements of given configuration.

Figure 1 shows individual thin-walled structural elements in which the temperature can change both along the z coordinate and within element sections for which the z coordinate is a normal because of the thermal action of the environment or other factors.

Before writing down the mathematical formulation of the problem, we give some definitions. Let D be the spatial region in which the distribution of the temperature T is sought. $D_j \in D$ is a subregion of the region D in which the temperature distribution is described by the traditional, and two-dimensional in this case, equations of thermal conductivity. In each region D_j we introduce an orthogonal coordinate system (z, x_j), $j = 1, 2, \ldots, N$. Note that the z coordinate is common to all D_j and the x_j are parallel to any section for which z is a

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